Recall that a (undirected) graph $G$ is defined as an ordered pair $(V, E)$ where $V$ is a finite non-empty set, and $E \subseteq V^2$ is a set edges (two-element subsets of $V$). The elements of $V$ are the vertices of $G$, and the elements of $E$ are the edges of $G$. It is convention to refer to $|V|$ as $n$ and $|E|$ as $m$.

An undirected bipartite graph $G$ is defined as an ordered pair $(V, E)$ where $V$ can be partitioned into two sets $A$ and $B$ such that $E \subseteq \{\{u, v\} \mid u \in A, v \in B\}$.

1. Prove by induction that, for any undirected graph $G = (V, E)$, the number of edges in $G$ is twice the sum of degrees of all vertices in $G$, i.e. $\sum_{v \in V} d(v) = 2m$.

2. Prove by induction that, for any undirected bipartite graph $G = (V, E)$ with bipartition $A$ and $B$, $\sum_{v \in A} d(v) = \sum_{v \in B} d(v) = m$.

3. Prove by induction that, for any binary string $s$ that begins with a 1 and ends with a 0, there is a 1 immediately before a 0 somewhere in $s$.

4. A rooted binary tree is full if every node has either zero or two children. Prove that any rooted full binary tree with $i$ internal nodes (those with at least one child) has $2i + 1$ total nodes.