The master Theorem
Let \( a \geq 1 \) and \( b > 1 \) be constants, let \( f(n) \) be a function, and let \( T(n) \) be defined on the nonnegative integers by the recurrence

\[
T(n) = aT(n/b) + f(n),
\]

where we interpret \( n/b \) to mean either \( \lfloor n/b \rfloor \) or \( \lceil n/b \rceil \). Then \( T(n) \) can be bounded asymptotically as follows.

1. If \( f(n) = O(n^{\log_b a - \epsilon}) \) for some constant \( \epsilon > 0 \), then \( T(n) = \Theta(n^{\log_b a}) \).

2. If \( f(n) = \Theta(n^{\log_b a}) \), then \( T(n) = \Theta(n^{\log_b a} \log n) \).

3. If \( f(n) = \Omega(n^{\log_b a + \epsilon}) \) for some constant \( \epsilon > 0 \), and if \( af(n/b) \leq cf(n) \) for some constant \( c < 1 \) and all sufficiently large \( n \), then \( T(n) = \Theta(f(n)) \).

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