CPS 570: Artificial Intelligence

Search

Instructor: Vincent Conitzer
Rubik’s Cube robot

- https://www.youtube.com/watch?v=iBE46R-fD6M
Search

• We have some actions that can change the state of the world
  – Change induced by an action is perfectly predictable

• Try to come up with a sequence of actions that will lead us to a goal state
  – May want to minimize number of actions
  – More generally, may want to minimize total cost of actions

• Do not need to execute actions in real life while searching for solution!
  – Everything perfectly predictable anyway
A simple example: traveling on a graph

start state  goal state
Searching for a solution
search tree nodes and states are not the same thing!
Full search tree

state = A, cost = 0

state = B, cost = 3

state = C, cost = 5

state = F, cost = 12

goal state!

state = A, cost = 7

state = D, cost = 3

state = E, cost = 7

state = F, cost = 11

goal state!
Changing the goal:
want to visit all vertices on the graph

need a different definition of a state
“currently at A, also visited B, C already”
large number of states: $n \times 2^{n-1}$
could turn these into a graph, but…
What would happen if the goal were to visit every location twice?
Key concepts in search

• Set of states that we can be in
  – Including an initial state…
  – … and goal states (equivalently, a goal test)

• For every state, a set of actions that we can take
  – Each action results in a new state
  – Typically defined by successor function
    • Given a state, produces all states that can be reached from it

• Cost function that determines the cost of each action (or path = sequence of actions)

• Solution: path from initial state to a goal state
  – Optimal solution: solution with minimal cost
8-puzzle

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goal state
8-puzzle
Generic search algorithm

- Fringe = set of nodes generated but not expanded

- fringe := \{ node with initial state \}

- loop:
  - if fringe empty, declare failure
  - choose and remove a node v from fringe
  - check if v’s state s is a goal state; if so, declare success
  - if not, expand v, insert resulting nodes into fringe

- Key question in search: Which of the generated nodes do we expand next?
Uninformed search

- Given a state, we only know whether it is a goal state or not
- Cannot say one nongoal state looks better than another nongoal state
- Can only traverse state space blindly in hope of somehow hitting a goal state at some point
  - Also called blind search
  - Blind does **not** imply unsystematic!
Breadth-first search
Properties of breadth-first search

- Nodes are expanded in the same order in which they are generated
  - Fringe can be maintained as a First-In-First-Out (FIFO) queue
- BFS is complete: if a solution exists, one will be found
- BFS finds a shallowest solution
  - Not necessarily an optimal solution
- If every node has $b$ successors (the branching factor), first solution is at depth $d$, then fringe size will be at least $b^d$ at some point
  - This much space (and time) required 😞
Depth-first search
Implementing depth-first search

- Fringe can be maintained as a Last-In-First-Out (LIFO) queue (aka. a stack)
- Also easy to implement recursively:
  - DFS(node)
    - If goal(node) return solution(node);
    - For each successor of node
      - Return DFS(successor) unless it is failure;
    - Return failure;
Properties of depth-first search

• Not complete (might cycle through nongoal states)
• If solution found, generally not optimal/shallowest
• If every node has b successors (the branching factor), and we search to at most depth m, fringe is at most bm
  – Much better space requirement 😊
  – Actually, generally don’t even need to store all of fringe
• Time: still need to look at every node
  – \( b^m + b^{m-1} + \ldots + 1 \) (for \( b > 1 \), \( O(b^m) \))
  – Inevitable for uninformed search methods…
Combining good properties of BFS and DFS

- **Limited depth DFS**: just like DFS, except never go deeper than some depth \( d \)
- **Iterative deepening DFS**:
  - Call limited depth DFS with depth 0;
  - If unsuccessful, call with depth 1;
  - If unsuccessful, call with depth 2;
  - Etc.
- Complete, finds shallowest solution
- Space requirements of DFS
- May seem wasteful timewise because replicating effort
  - Really not that wasteful because *almost all effort at deepest level*
  - \( db + (d-1)b^2 + (d-2)b^3 + \ldots + 1b^d \) is \( O(b^d) \) for \( b > 1 \)
Let’s start thinking about cost

- BFS finds shallowest solution because always works on shallowest nodes first
- Similar idea: always work on the lowest-cost node first (uniform-cost search)
- Will find optimal solution (assuming costs increase by at least constant amount along path)
- Will often pursue lots of short steps first
- If optimal cost is $C$, and cost increases by at least $L$ each step, we can go to depth $C/L$
- Similar memory problems as BFS
  - Iterative lengthening DFS does DFS up to increasing costs
Searching backwards from the goal

- Sometimes can search backwards from the goal
  - Maze puzzles
  - Eights puzzle
  - Reaching location F
  - What about the goal of “having visited all locations”?

- Need to be able to compute predecessors instead of successors

- What’s the point?
Predecessor branching factor can be smaller than successor branching factor

• Stacking blocks:
  – only action is to add something to the stack

\[ \text{In hand: A, B, C} \]
\[ \text{Start state} \]

\[ \text{In hand: nothing} \]
\[ \text{Goal state} \]

*We’ll see more of this...*
Bidirectional search

• Even better: search from both the start and the goal, in parallel!

• If the shallowest solution has depth $d$ and branching factor is $b$ on both sides, requires only $O(b^{d/2})$ nodes to be explored!
Making bidirectional search work

• Need to be able to figure out whether the fringes intersect
  – Need to keep at least one fringe in memory…

• Other than that, can do various kinds of search on either tree, and get the corresponding optimality etc. guarantees

• Not possible (feasible) if backwards search not possible (feasible)
  – Hard to compute predecessors
  – High predecessor branching factor
  – Too many goal states
Repeated states can cause incompleteness or enormous runtimes.

Can maintain list of previously visited states to avoid this:
- If new path to the same state has greater cost, don’t pursue it further.
- Leads to time/space tradeoff.

“Algorithms that forget their history are doomed to repeat it” [Russell and Norvig]
Informed search

• So far, have assumed that no nongoal state looks better than another

• Unrealistic
  – Even without knowing the road structure, some locations seem closer to the goal than others
  – Some states of the 8s puzzle seem closer to the goal than others

• Makes sense to expand closer-seeming nodes first
Heuristics

- Key notion: heuristic function $h(n)$ gives an estimate of the distance from $n$ to the goal
  - $h(n)=0$ for goal nodes
- E.g. straight-line distance for traveling problem

![Graph]

- Say: $h(A) = 9$, $h(B) = 8$, $h(C) = 9$, $h(D) = 6$, $h(E) = 3$, $h(F) = 0$
- We’re adding something new to the problem!
- Can use heuristic to decide which nodes to expand first
Greedy best-first search

- **Greedy best-first search**: expand nodes with lowest $h$ values first

- Rapidly finds the optimal solution!
- Does it always?

![Diagram of search tree with states and costs](image_url)
A bad example for greedy

• Say: \( h(A) = 9, h(B) = 5, h(D) = 6, h(E) = 3, h(F) = 0 \)

• Problem: greedy evaluates the promise of a node only by how far is left to go, does not take cost occurred already into account
A*

- Let \( g(n) \) be cost incurred already on path to \( n \)
- Expand nodes with lowest \( g(n) + h(n) \) first

- Say: \( h(A) = 9, h(B) = 5, h(D) = 6, h(E) = 3, h(F) = 0 \)

- Note: if \( h=0 \) everywhere, then just uniform cost search
Admissibility

• A heuristic is **admissible** if it never overestimates the distance to the goal
  – If $n$ is the optimal solution reachable from $n'$, then $g(n) \geq g(n') + h(n')$

• Straight-line distance is admissible: can’t hope for anything better than a straight road to the goal

• Admissible heuristic means that $A^*$ is always optimistic
Optimality of A*

• If the heuristic is admissible, A* is optimal (in the sense that it will never return a suboptimal solution)

• Proof:
  – Suppose a suboptimal solution node n with solution value \( C > C^* \) is about to be expanded (where \( C^* \) is optimal)
  – Let \( n^* \) be an optimal solution node (perhaps not yet discovered)
  – There must be some node \( n' \) that is currently in the fringe and on the path to \( n^* \)
  – We have \( g(n) = C > C^* = g(n^*) \geq g(n') + h(n') \)
  – But then, \( n' \) should be expanded first (contradiction)
A* is not complete (in contrived examples)

- No optimal search algorithm can succeed on this example (have to keep looking down the path in hope of suddenly finding a solution)
Consistency

- A heuristic is **consistent** if the following holds: if one step takes us from \( n \) to \( n' \), then \( h(n) \leq h(n') + \text{cost of step from } n \text{ to } n' \)
  - Similar to triangle inequality
  - Equivalently, \( g(n)+h(n) \leq g(n')+h(n') \)
- Implies admissibility
- It’s strange for an admissible heuristic not to be consistent!
  - Suppose \( g(n)+h(n) > g(n')+h(n') \). Then at \( n' \), we know the remaining cost is at least \( h(n)-(g(n')-g(n)) \), otherwise the heuristic wouldn’t have been admissible at \( n \). But then we can safely increase \( h(n') \) to this value.
A* is optimally efficient

- A* is **optimally efficient** in the sense that any other optimal algorithm must expand at least the nodes A* expands, if the heuristic is consistent.

Proof:

- Besides solution, A* expands exactly the nodes with \( g(n) + h(n) < C^* \) (due to consistency).
  - Assuming it does not expand non-solution nodes with \( g(n) + h(n) = C^* \)
  - Any other optimal algorithm must expand at least these nodes (since there may be a better solution there).

Note: This argument assumes that the other algorithm uses the same heuristic \( h \).
A* and repeated states

• Suppose we try to avoid repeated states
• Ideally, the second (or third, …) time that we reach a state the cost is at least as high as the first time
  – Otherwise, have to update everything that came after
• This is guaranteed if the heuristic is consistent
Proof

• Suppose \( n \) and \( n' \) correspond to same state, \( n' \) is cheaper to reach, but \( n \) is expanded first

• \( n' \) cannot have been in the fringe when \( n \) was expanded because \( g(n') < g(n) \), so
  \[ g(n') + h(n') < g(n) + h(n) \]

• So \( n' \) is generated (eventually) from some other node \( n'' \) currently in the fringe, after \( n \) is expanded
  \[ g(n) + h(n) \leq g(n'') + h(n'') \]

• Combining these, we get
  \[ g(n') + h(n') < g(n'') + h(n''), \text{ or equivalently} \]
  \[ h(n'') > h(n') + \text{cost of steps from } n'' \text{ to } n' \]
  • Violates consistency
Iterative Deepening A*

• One big drawback of A* is the space requirement: similar problems as uniform cost search, BFS

• **Limited-cost depth-first A***: some cost cutoff \( c \), any node with \( g(n) + h(n) > c \) is not expanded, otherwise DFS

• IDA* gradually increases the cutoff of this

• Can require lots of iterations
  – Trading off space and time…
  – **RBFS** algorithm reduces wasted effort of IDA*, still linear space requirement
  – **SMA*** proceeds as A* until memory is full, then starts doing other things
More about heuristics

- One heuristic: number of misplaced tiles
- Another heuristic: sum of **Manhattan distances** of tiles to their goal location
  - Manhattan distance = number of moves required if no other tiles are in the way
- Admissible? Which is better?
- Admissible heuristic $h_1$ **dominates** admissible heuristic $h_2$ if $h_1(n) \geq h_2(n)$ for all $n$
  - Will result in fewer node expansions
- “Best” heuristic of all: solve the remainder of the problem optimally with search
  - Need to worry about computation time of heuristics…
Designing heuristics

• One strategy for designing heuristics: relax the problem (make it easier)

• “Number of misplaced tiles” heuristic corresponds to relaxed problem where tiles can jump to any location, even if something else is already there

• “Sum of Manhattan distances” corresponds to relaxed problem where multiple tiles can occupy the same spot

• Another relaxed problem: only move 1,2,3,4 into correct locations

• The ideal relaxed problem is
  – easy to solve,
  – not much cheaper to solve than original problem

• Some programs can successfully automatically create heuristics
Macro-operators

• Perhaps a more human way of thinking about search in the eights puzzle:

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sequence of operations = macro-operation

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• We swapped two adjacent tiles, and rotated everything

• Can get all tiles in the right order this way
  – Order might still be rotated in one of eight different ways; could solve these separately

• Optimality?

• Can AI think about the problem this way? Should it?