Artificial Intelligence

Decision theory

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Risk attitudes

• Which would you prefer?
  – A lottery ticket that pays out $10 with probability .5 and $0 otherwise, or
  – A lottery ticket that pays out $3 with probability 1

• How about:
  – A lottery ticket that pays out $100,000,000 with probability .5 and $0 otherwise, or
  – A lottery ticket that pays out $30,000,000 with probability 1

• Usually, people do not simply go by expected value

• An agent is risk-neutral if she only cares about the expected value of the lottery ticket

• An agent is risk-averse if she always prefers the expected value of the lottery ticket to the lottery ticket
  – Most people are like this

• An agent is risk-seeking if she always prefers the lottery ticket to the expected value of the lottery ticket
Decreasing marginal utility

• Typically, at some point, having an extra dollar does not make people much happier (decreasing marginal utility)
Maximizing expected utility

- Lottery 1: get $1500 with probability 1
  - gives expected utility 2
- Lottery 2: get $5000 with probability .4, $200 otherwise
  - gives expected utility $0.4 \times 3 + 0.6 \times 1 = 1.8$
  - (expected amount of money = $0.4 \times 5000 + 0.6 \times 200 = 2120 > 1500$)
- So: maximizing expected utility is consistent with risk aversion
Different possible risk attitudes under expected utility maximization

- **Green** has decreasing marginal utility → risk-averse
- **Blue** has constant marginal utility → risk-neutral
- **Red** has increasing marginal utility → risk-seeking
- **Grey**’s marginal utility is sometimes increasing, sometimes decreasing → neither risk-averse (everywhere) nor risk-seeking (everywhere)
Acting optimally over time

- **Finite** number of periods:
  - Overall utility = sum of rewards in individual periods

- **Infinite** number of periods:
  - … are we just going to add up the rewards over infinitely many periods?
    - Always get infinity!
  - (Limit of) *average* payoff: \( \lim_{n \to \infty} \sum_{1 \leq t \leq n} r(t)/n \)
    - Limit may not exist…
  - **Discounted** payoff: \( \sum_t \delta^t r(t) \) for some \( \delta < 1 \)

- Interpretations of discounting:
  - Interest rate \( r \): \( \delta = 1/(1+r) \)
  - World ends with some probability \( 1-\delta \)

- Discounting is mathematically convenient