Fall 2012 - Artificial Intelligence Qual

CPS 570
1 Informed Search (12 points)

For each of the followings questions, you will be given two heuristics and asked about how their properties are preserved when they are combined in various ways. Assume that the functions $f$ and $g$ referenced below can return any non-negative rational number. You must justify your answer to get full credit.

a) Suppose $f(x)$ and $g(x)$ are both admissible. Is $0.7f(x) + 0.1g(x)$ admissible? (4 points)

b) Suppose $f(x)$ and $g(x)$ are both admissible. Is $f(x)/g(x)$ admissible? (4 points)
c) Suppose $f(x)$ and $g(x)$ are both consistent. Is $\max\{0, f(x) - g(x)\}$ consistent? (Recall that a consistent heuristic, also called a \textit{monotone} heuristic, has the property that $h(n) \leq c(n, p) + h(p)$, where $c(n, p)$ is the cost of reaching $p$ from $n$.) (4 points)
2 Game Tree Search (12 points)

Suppose you try a simple scheme to speed up game tree search (with alpha-beta pruning) on a machine with 2 cores. Your scheme assigns cores to siblings in the game tree then combines the results when both results are returned. Assume that all nodes have an even number of children. Demonstrate, using a simple example with clearly labeled node values, how this scheme will fail to achieve linear speedup due to failure to prune nodes that would be pruned in a single-core implementation.
3 Logic (12 points)

Consider the following statements:

- Everybody who lives in North Carolina likes BBQ and likes basketball.
- Jim likes BBQ.
- Jim does not like basketball.

a) Convert the above statements to first order logic. (4 points)

b) Convert your answer to the previous part into the appropriate form (CNF) for use in a resolution proof. (4 points)
c) Provide a *resolution proof* that Jim does not live in North Carolina. (4 points)
4 Probability and Bayes Nets (16 points)

For this question, we will use the follow probability distribution:

<table>
<thead>
<tr>
<th>Atomic Event</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(\overline{abc})$</td>
<td>$\frac{1}{8}$</td>
</tr>
<tr>
<td>$P(\overline{abc})$</td>
<td>$\frac{1}{8}$</td>
</tr>
<tr>
<td>$P(\overline{abc})$</td>
<td>$\frac{1}{6}$</td>
</tr>
<tr>
<td>$P(\overline{abc})$</td>
<td>$\frac{1}{12}$</td>
</tr>
<tr>
<td>$P(\overline{abc})$</td>
<td>$\frac{3}{16}$</td>
</tr>
<tr>
<td>$P(\overline{abc})$</td>
<td>$\frac{3}{16}$</td>
</tr>
<tr>
<td>$P(ab\overline{c})$</td>
<td>$\frac{1}{12}$</td>
</tr>
<tr>
<td>$P(abc)$</td>
<td>$\frac{1}{24}$</td>
</tr>
</tbody>
</table>

a) Prove that $C$ is conditionally independent of $A$ given $B$. *Hint: This is a little tedious because you need to compute a lot of things to prove this, but it's not hard. The fractions were chosen to make the arithmetic simple. You may want to use the back of this sheet for additional space.* (8 points)
b) Provide the conditional probability tables for a Bayesian network for this distribution where variable $C$ has $B$ as its only parent, variable $B$ has variable $A$ as its only parent, and variable $A$ has no parents. (8 points)
5 CSPs (16 points)

Refer to a set nodes in a graph as a cycle cutset (cutset for short, in this problem) if removal of these nodes transforms the graph into a tree (or forest of trees). For a simple example of a cutset, consider a set of \( n \) nodes connected in a loop such that there is an edge between \( v_i \) and \( v_{i+1} \) for \( 1 \leq i \leq n - 1 \), and an edge between \( v_n \) and \( v_1 \). In this case, any single node is a cutset.

Now consider the following algorithm for solving a CSP: In the graph representation of the CSP, identify a cutset. For each setting of the variables in the cutset, try to find a consistent setting for the variables in the tree(s) (where consistency refers both to consistency within the tree and consistency with the settings of the variables in the cutset) until either a satisfying assignment for the entire problem is found, or until all settings of the variables in the cutset are tried.

Suppose there are \( n \) variables which can take on \( m \) different values, and suppose that your cutset is of size \( c \). Provide and justify an \( O() \) expression for the run time of this algorithm in terms of \( m \), \( n \), and \( c \).
6 Value of Information (16 points)

For this problem, you will prove that the value of information is not additive. More precisely: Consider an action variable $A$, two evidence variables, $E_1$ and $E_2$, a state variable $S$, a utility function defined over $S$, a joint distribution $P(E_1, E_2)$, and a conditional distribution $P(S|E_1, E_2)$. Use these to compute the value of perfect information: $VPI(E_1), VPI(E_2)$ and $VPI(E_1, E_2)$. You should set things up so that $VPI(E_1) + VPI(E_2) \neq VPI(E_1, E_2)$. Hint: It may seem like a long problem, but you can simplify things a lot with your choice of probability distributions.
Consider a problem where you have a choice between earning $1 per day, every day *forever*, or a one-time payment of $1000.

a) Formulate this problem as an MDP. Indicate clearly what the states and rewards are for this MDP. (4 points)

b) Derive a value of the discount factor for which one is indifferent between the one time payoff and the continuing payoffs. (6 points)
c) Solve for a value of the reward for which one should be indifferent between the one time payoff and the continuing payoffs when $\gamma = 0.9$. (6 points)